Consider two straight beams with thin-walled circular cross-sections of radius R and wall thickness t, where $R \gg t$. One beam has an open circular cross-section with a small gap at the bottom (see Figure 1), while the other has a closed circular cross-section with no gap (see Figure 2). A horizontal shear force of magnitude V_y is applied at the origin along the y-axis. The origin is chosen as the centroid of the circular cross-section.

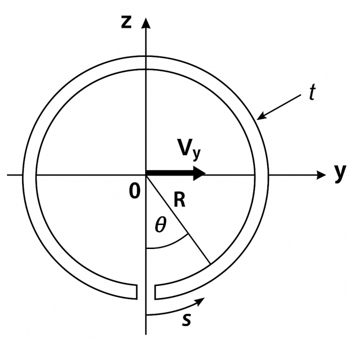

Figure 1: Open circular cross-section with a small gap at the bottom

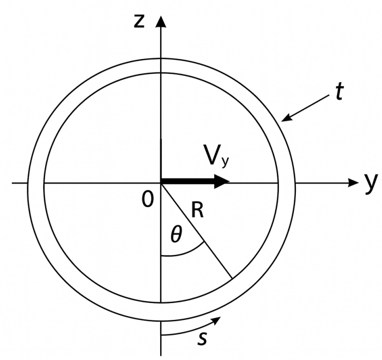

Figure 2: Closed circular cross-section without a gap

Answer the following questions:

1. Determine the distribution of flexural shear flow for each cross-section due to this shear force.
2. For the open section, can you use the equation $M_{q_1} = 2Aq_1$, where A is the enclosed area and q_1 is the shear flow for the open section, to calculate the moment due to shear flow M_{q_1}? Why or why not?
3. What is the location of the shear center for the open section and the closed section? What effect does making the cut at the bottom of the circular cross-section have on the location of the shear center?

Solution

1. Determine the distribution of flexural shear flow for each cross-section due to this shear force.

   For both sections, the relations of differentials apply:

   $$ds = R\,d\theta, \quad dA = t\,ds = Rt\,d\theta$$

   For the open section, shear flow q_1 is given by:

   $$q_1 = -\frac{V_y Q_z}{I_z} = -\frac{V_y \int_A y\,dA}{\int_A y^2\,dA}$$

Substitute $y = R\sin\theta$ and $dA = Rt\,d\theta$. The numerator is integrated from 0 to θ because shear flow at a point depends on the first moment of the area above that point:

$$q_1 = -\frac{V_y \int_0^{\theta} R\sin\theta\, Rt\,d\theta}{\int_0^{2\pi} R^2 \sin^2\theta\, Rt\,d\theta} = \frac{V_y R^2 t(\cos\theta - 1)}{R^3 t \int_0^{2\pi} \sin^2\theta\,d\theta}$$

where

$$\int_0^{2\pi} \sin^2\theta\,d\theta = \pi$$

Therefore, shear flow in the open section q_1 is

$$q_1 = \frac{V_y(\cos\theta - 1)}{R\pi}$$

Now consider the closed cross-section. To close the open section, we conceptually add a constant shear flow q_0. Let the total shear flow in the closed section be q_2, so that:

$$q_2 = q_1 + q_0$$

Assume the shear force is applied at the shear center. By definition of the shear center, applying a shear force through it does not produce any twist. Therefore, the twist rate is zero:

$$\theta = \frac{1}{2GA}\oint \frac{q_2}{t}\,ds = \frac{1}{2GAt}\oint (q_1 + q_0)\,ds = 0$$

This leads to:

$$\oint (q_1 + q_0)\,ds = \int_0^{2\pi} \left(\frac{V_y(\cos\theta - 1)}{R\pi} + q_0\right) R\,d\theta = 0$$

Split the integral:

$$\frac{V_y}{\pi}\int_0^{2\pi} (\cos\theta - 1)\,d\theta + 2\pi R q_0 = 0$$

where

$$\int_0^{2\pi} (\cos\theta - 1)\,d\theta = -2\pi$$

So we get:

$$\frac{V_y}{\pi}(-2\pi) + 2\pi R q_0 = 0$$

Solving for q_0:

$$q_0 = \frac{V_y}{\pi R}$$

Therfore, the shear flow for the closed section q_2 is:

$$q_2 = q_1 + q_0 = \frac{V_y(\cos\theta - 1)}{\pi R} + \frac{V_y}{\pi R} = \frac{V_y \cos\theta}{\pi R}$$

2. For the open section, can you use the equation $M_{q_1} = 2Aq_1$, where A is the enclosed area and q_1 is the shear flow for the open section, to calculate the moment due to shear flow M_{q_1}? Why or why not?

   No, the equation $M_{q_1} = 2Aq_1$ cannot be used because the equation assumes a constant shear flow q_1 in the closed section. This equation is only applicable to problems where the thin-walled section does not resist bending, such as stringer-web sections. It does not apply to any problems where thin-walled section also resist bending. For the open section in this problem, the shear flow is not constant because it is a function of θ, as derived in Part 1:

$$q_1 = \frac{V_y(\cos\theta - 1)}{R\pi}$$

   To compute the moment due to shear flow in the open section, integrate the differential moment:

$$dM_{q_1} = Rq_1\,ds = R^2 q_1\,d\theta$$

   Therefore, the total moment is:

$$M_{q_1} = \int_0^{2\pi} R^2 q_1\,d\theta$$

   If q_1 was a constant, it could be taken out of the integral:

$$M_{q_1} = q_1 \int_0^{2\pi} R^2\,d\theta = 2\pi R^2 q_1$$

   Since the area of the circle is $A = \pi R^2$, this becomes:

$$M_{q_1} = 2Aq_1$$

   which is the equation mentioned by the problem.

   However, because q_1 is not constant, it cannot be pulled out of the integral, so $M_{q_1} \neq 2Aq_1$. Instead, we have

$$M_{q_1} = \int_0^{2\pi} R^2 q_1\,d\theta = \int_0^{2\pi} R^2 \frac{V_y(\cos\theta - 1)}{R\pi}\,d\theta = \frac{V_y R}{\pi}\int_0^{2\pi} (\cos\theta - 1)\,d\theta$$

   Evaluate the integral:

$$\int_0^{2\pi} (\cos\theta - 1)\,d\theta = -2\pi$$

   So the moment due to shear flow becomes:

$$M_{q_1} = \frac{V_y R}{\pi}(-2\pi) = -2V_y R$$

   This result clearly cannot be obtained using $M_{q_1} = 2Aq_1$.

3. What is the location of the shear center for the open section and the closed section? What effect does making the cut at the bottom of the circular cross-section have on the location of the shear center?

   For the open section, the moment due to shear flow is calculated as:

$$M_{q_1} = \frac{V_y R}{\pi}(-2\pi) = -2V_y R$$

   Let the location of the shear center be (y_{sc}, z_{sc}) where $y_{sc} = 0$ because the open section is symmetric about z-axis. Now apply shear force V_y at the shear center instead of the origin. We obtain the moment due to applied load:

$$M_{V_y} = -V_y z_{sc}$$

   We know that the moment due to shear flow is equivalent to the moment due to applied load at the origin:

$$-2V_y R = -V_y z_{sc}$$

   Therefore, the z coordinate of the shear center z_{sc} is:

$$z_{sc} = 2R$$

   This means the location of the shear center for the open section is $(0, 2R)$, which is outside the circle on the opposite side of the circular cross-section.

   For the closed section, the moment due to shear flow is calculated as:

$$M_{q_2} = \int_0^{2\pi} R^2 q_2 \, d\theta = \int_0^{2\pi} R^2 \frac{V_y \cos\theta}{\pi R} \, d\theta = \frac{V_y R}{\pi} \int_0^{2\pi} \cos\theta \, d\theta = 0$$

   Similar to the open section, the resulting moment due to the applied load is also:

$$M_{V_y} = -V_y z_{sc}$$

   We know that the moment due to shear flow must equal the moment caused by applying the shear force at the origin:

$$0 = -V_y z_{sc}$$

   which gives the z-coordinate of the shear center:

$$z_{sc} = 0$$

   This means the location of the shear center for the closed section is $(0, 0)$, which lies exactly at the center of the circular cross-section.

   Based on the location of the shear center of the open and closed section, the shear center shifts a distance of $2R$ away from the opening when we make a cut at the bottom of the closed section.