Dear Dr. Orzuri Rique,

This is Hongzhou Zhai from Nanjing University of Aeronautics and Astronautics, China.

I recently read your interesting paper "Constitutive modeling for time- and temperature-dependent behavior of Composites, 10.1016/j.compositesb.2019.107726" that reinforces my knowledge on multiscale analysis of composite materials. However, I do not understand some details and want to learn more. Therefore, I am writing this letter to ask for your direction.

In Page 2, the constitutive equation for an anisotropic linear thermoviscoelastic material and an expression for computing time- and temperature-dependent thermal expansion coefficient (CTE) are given as

$$\sigma_{ij}(t)=\int_0^t\left\{C_{ijkl}(T,t-\tau)\frac{\partial}{\partial\tau}\left[\varepsilon_{kl}(\tau)\right]-\beta_{ij}(T,t-\tau)\frac{\partial}{\partial\tau}\left[\theta(\tau)\right]\right\}d\tau \quad \text{(A1)}$$

$$\alpha_{ij}(T,t)=-\left(C_{ijkl}(T,t)\right)^{-1}\beta(T,t) \quad \text{(A2)}$$

where $\sigma_{ij}(t)$ are instantaneous stress components, T is the temperature, t and τ are time, $C_{ijkl}(T,t)$ is the stress relaxation stiffness which is a function of time and temperature, $\varepsilon_{kl}(t)$ are strain components, $\beta_{ij}(T,t)$ is the instantaneous thermal stress tensor, $\theta(t)$ is the temperature change from the stress free starting temperature and $\alpha_{ij}(T,t)$ is the CTE. <u>(A2) is a multiplication equation.</u> <u>Is it reasonable to compute thermal expansion coefficient with Eq. (A2)?</u>
The constitutive equation[1] for an anisotropic linear thermoviscoelastic material also yields

$$\sigma_{ij}(T,t)=\int_0^t\left\{C_{ijkl}(T,t-\tau)\frac{\partial}{\partial\tau}\left[\varepsilon_{kl}(\tau)\right]-C_{ijkl}(T,t-\tau)\frac{\partial}{\partial\tau}\left[\alpha_{kl}(T,\tau)\cdot\theta(\tau)\right]\right\}d\tau \quad \text{(A3)}$$

The strain can be divided into mechanical strain $\varepsilon_{kl}^{mec}(t)$ and thermal expansion strain $\varepsilon_{kl}^{th}(t)$, given as

$$\varepsilon_{kl}(t)=\varepsilon_{kl}^{mec}(t)+\varepsilon_{kl}^{th}(t) \quad \text{(A4)}$$

Comparing Eq. (A1) to Eq. (A3), the thermal stress σ_{ij}^{th} in the thermo-viscoelastic composite yields

$$\sigma_{ij}^{th}(T,t)=\int_0^t\left\{-C_{ijkl}(T,t-\tau)\frac{\partial}{\partial\tau}\left[\alpha_{kl}(T,\tau)\cdot\theta(\tau)\right]\right\}d\tau=\int_0^t\left\{-\beta_{ij}(T,t-\tau)\frac{\partial}{\partial\tau}\left[\theta(\tau)\right]\right\}d\tau \quad \text{(A5)}$$

(A5) is a convolution equation, differing significantly from (A2). Equations (A2) and (A5) are equivalent only when α_{kl} is time-independent. This result is inconsistent to yours written in the paper, and makes me very confusing.

In a word, I guess equation (A2) should presenting a convolution format, as

$$\alpha_{ij}(T,t)=-J_{ijkl}(T,t)*\frac{\partial}{\partial t}\left[\beta(T,t)\right] \quad \text{(A6)}$$

where $J_{ijkl}(T,t)$ is the creep stiffness of the thermoviscoelastic material.

Reference

[1] P. W. Chung, K. K. Tamma, and R. R. Namburu, “A finite element thermo-viscoelastic creep approach for heterogeneous structures with dissipative correctors,” *Finite Elem. Anal. Des.*, vol. 35, pp. 279–313, 2000.

I think maybe there are some points I am missing or do not understanding well. Could you please teach me?

Best regards

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